CHAPTER 5
DECIMALS

The origin and meaning of the word "decimal" were discussed in chapter 1 of this course. Also discussed in chapter 1 were the concept of place value and the use of the number ten as the base for our number system. Another term which is frequently used to denote the base of a number system is RADIX. For example, two is the radix of the binary system and ten is the radix of the decimal system. The radix of a number system is always equal to the number of different digits used in the system. For example, the decimal system, with radix ten, has ten digits: 0 through 9.

DECIMAL FRACTIONS

A decimal fraction is a fraction whose denominator is 10 or some power of 10, such as 100, 1,000, or 10,000. Thus, $\frac{7}{10}$, $\frac{12}{100}$, and $\frac{215}{1000}$ are decimal fractions. Decimal fractions have special characteristics that make computation much simpler than with other fractions.

Decimal fractions complete our decimal system of numbers. In the study of whole numbers, we found that we could proceed to the left from the units place, tens, hundreds, thousands, and on indefinitely to any larger place value, but the development stopped with the units place. Decimal fractions complete the development so that we can proceed to the right of the units place to any smaller number indefinitely.

Figure 5-1 (A) shows how decimal fractions complete the system. It should be noted that as we proceed from left to right, the value of each place is one-tenth the value of the preceding place, and that the system continues uninterrupted with the decimal fractions.

Figure 5-1 (B) shows the system again, this time using numbers. Notice in (A) and (B) that the units place is the center of the system and that the place values proceed to the right or left of it by powers of ten. Ten on the left is balanced by tenths on the right, hundreds by hundredths, thousands by thousandths, etc.

Notice that 1/10 is one place to the right of the units digit, 1/100 is two places to the right, etc. (See fig. 5-1.) If a marker is placed after the units digit, we can decide whether a decimal digit is in the tenths, hundredths, or thousandths position by counting places to the right of the marker. In some European countries, the marker is a comma; but in the English-speaking countries, the marker is the DECIMAL POINT.

Thus, $\frac{3}{10}$ is written 0.3. To write $\frac{3}{100}$ it is necessary to show that 3 is in the second place to the right of the decimal point, so a zero is inserted in the first place. Thus, $\frac{3}{100}$ is written 0.03. Similarly, $\frac{3}{1000}$ can be written by inserting zeros in the first two places to the right of the decimal point. Thus, $\frac{3}{1000}$ is written 0.003. In the number 0.3, we say that 3 is in the first decimal place; in 0.03, 3 is in the second decimal place; and in 0.003, 3 is in the third decimal place. Quite frequently decimal fractions are simply called decimals when written in this shortened form.

WRITING DECIMALS

Any decimal fraction may be written in the shortened form by a simple mechanical process. Simply begin at the right-hand digit of the numerator and count off to the left as many places as there are zeros in the denominator. Place the decimal point to the left of the last digit counted. The denominator may then be disregarded. If there are not enough digits, as many place-holding zeros as are necessary are added to the left of the left-hand digit in the numerator.

Thus, in $\frac{23}{10000}$, beginning with the digit 3, we count off four places to the left, adding two 0's as we count, and place the decimal point to the extreme left. (See fig. 5-2.) Either form is read "twenty-three ten-thousandths."

When a decimal fraction is written in the shortened form, there will always be as many decimal places in the shortened form as there

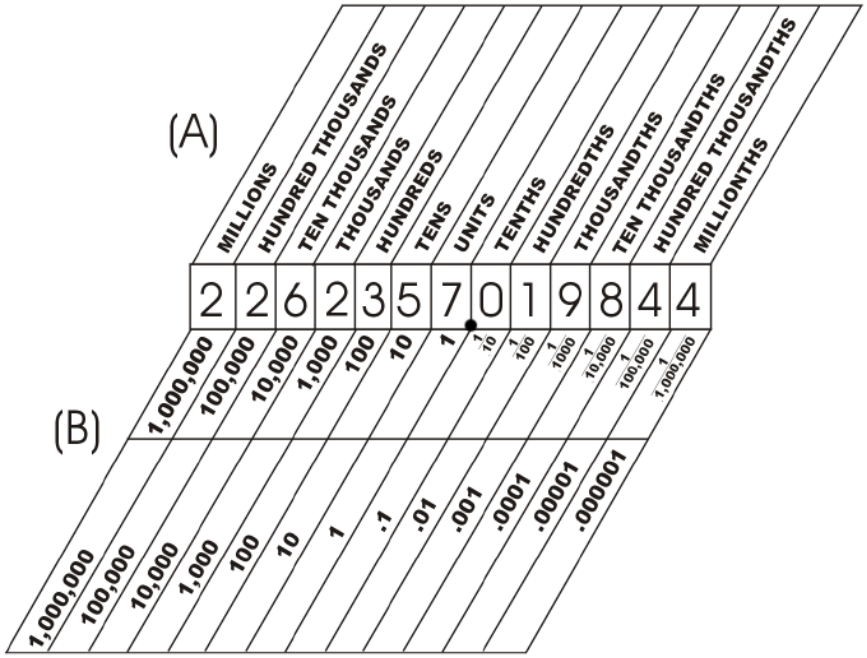

Figure 5-1.–Place values including decimals.

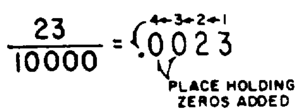

Figure 5-2.–Conversion of a decimal fraction to shortened form.

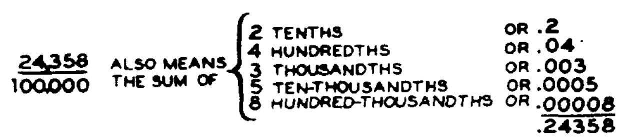

Figure 5-3.–Steps in the conversion of a decimal fraction to shortened form.

are zeros in the denominator of the fractional form.

Figure 5-3 shows the fraction $\frac{24358}{100000}$ and what is meant when it is changed to the shortened form. This figure is presented to show further that each digit of a decimal fraction holds a certain position in the digit sequence and has a particular value.

By the fundamental rule of fractions, it should be clear that $\frac{5}{10} = \frac{50}{100} = \frac{500}{1000}$. Writing the same values in the shortened way, we have $0.5 = 0.50 = 0.500$. In other words, the value of a decimal is not changed by annexing zeros at the right-hand end of the number. This is not true of whole numbers. Thus, 0.3, 0.30, and 0.300 are equal but 3, 30, and 300 are not equal. Also notice that zeros directly after the decimal point do change values. Thus 0.3 is not equal to either 0.03 or 0.003.

Decimals such as 0.125 are frequently seen. Although the 0 on the left of the decimal point is not required, it is often helpful. This is particularlytrue in an expression such as $32 \div 0.1$. In this expression, the lower dot of the division symbol must not be crowded against the decimal point; the 0 serves as an effective spacer. If any doubt exists concerning the clarity of an expression such as .125, it should be written as 0.125.

Practice problems. In problems 1 through 4, change the fractions to decimals. In problems 5 through 8, write the given numbers as decimals:

1. 8/100	5. Four hundredths
2. 5/1000	6. Four thousandths
3. 43/1000	7. Five hundred one ten-thousandths
4. 32/10000	8. Ninety-seven thousandths

Answers:

1. 0.08	5. 0.04
2. 0.005	6. 0.004
3. 0.043	7. 0.0501
4. 0.0032	8. 0.097

READING DECIMALS

To read a decimal fraction in full, we read both its numerator and denominator, as in reading common fractions. To read 0.305, we read "three hundred five thousandths." The denominator is always 1 with as many zeros as decimal places. Thus the denominator for 0.14 is 1 with two zeros, or 100. For 0.003 it is 1,000; for 0.101 it is 1,000; and for 0.3 it is 10. The denominator may also be determined by counting off place values of the decimal. For 0.13 we may think "tenths, hundredths" and the fraction is in hundredths. In the example 0.1276 we may think "tenths, hundredths, thousandths, ten-thousandths." We see that the denominator is 10,000 and we read the fraction "one thousand two hundred seventy-six ten-thousandths."

A whole number with a fraction in the form of a decimal is called a MIXED DECIMAL. Mixed decimals are read in the same manner as mixed numbers. We read the whole number in the usual way followed by the word "and" and then read the decimal. Thus, 160.32 is read "one hundred sixty and thirty-two hundredths." The word "and" in this case, as with mixed numbers, means plus. The number 3.2 means three plus two tenths.

It is also possible to have a complex decimal. A COMPLEX DECIMAL contains a common fraction. The number $0.3\frac{1}{3}$ is a complex decimal and is read "three and one-third tenths." The number $0.87\frac{1}{2}$ means $87\frac{1}{2}$ hundredths. The common fraction in each case forms a part of the last or right-hand place.

In actual practice when numbers are called out for recording, the above procedure is not used. Instead, the digits are merely called out in order with the proper placing of the decimal point. For example, the number 216.003 is read, "two one six point zero zero three." The number 0.05 is read, "zero point zero five."

EQUIVALENT DECIMALS

Decimal fractions may be changed to equivalent fractions of higher or lower terms, as is the case with common fractions. If each decimal fraction is rewritten in its common fraction form, changing to higher terms is accomplished by multiplying both numerator and denominator by 10, or 100, or some higher power of 10. For example, if we desire to change $\frac{5}{10}$ to hundredths, we may do so by multiplying both numerator and denominator by 10. Thus,

$$\frac{5}{10} = \frac{50}{100}$$

In the decimal form, the same thing may be accomplished by simply annexing a zero. Thus,

$$0.5 = 0.50$$

Annexing a 0 on a decimal has the same effect as multiplying the common fraction form of the decimal by 10/10. This is an application of the fundamental rule of fractions. Annexing two 0's has the same effect as multiplying the common fraction form of the decimal by 100/100; annexing three 0's has the same effect as multiplying by 1000/1000; etc.

REDUCTION TO LOWER TERMS

Reducing to lower terms is known as ROUND-OFF, or simply ROUNDING, when dealing with decimal fractions. If it is desired to reduce 6.3000 to lower terms, we may simply drop as many end zeros as necessary since this is equivalent to dividing both terms of the fraction by some power of ten. Thus, we see that 6.3000 is the same as 6.300, 6.30, or 6.3.

It is frequently necessary to reduce a number such as 6.427 to some lesser degree of precision. For example, suppose that 6.427 is to be rounded to the nearest hundredth. The question to be decided is whether 6.427 is closer to 6.42 or 6.43. The best way to decide this question is to compare the fractions 420/1000, 427/1000, and 430/1000. It is obvious that 427/1000 is closer to 430/1000, and 430/1000 is equivalent to 43/100; therefore we say that 6.427, correct to the nearest hundredth, is 6.43.

A mechanical rule for rounding off can be developed from the foregoing analysis. Since the digit in the tenths place is not affected when we round 6.427 to hundredths, we may limit our attention to the digits in the hundredths and thousandths places. Thus the decision reduces to the question whether 27 is closer to 20 or 30. Noting that 25 is halfway between 20 and 30, it is clear that anything greater than 25 is closer to 30 than it is to 20.

In any number between 20 and 30, if the digit in the thousandths place is greater than 5, then the number formed by the hundredths and thousandths digits is greater than 25. Thus we would round the 27 in our original problem to 30, as far as the hundredths and thousandths digits are concerned. This result could be summarized as follows: When rounding to hundredths, if the digit in the thousandths place is greater than 5, increase the digit in the hundredths place by 1 and drop the digit in the thousandths place.

The digit in the thousandths place may be any one of the ten digits, 0 through 9. If these ten digits are split into two groups, one composed of the five smaller digits (0 through 4) and the other composed of the five larger digits, then 5 is counted as one of the larger digits. Therefore, the general rule for rounding off is stated as follows: If the digit in the decimal place to be eliminated is 5 or greater, increase the digit in the next decimal place to the left by 1. If the digit to be eliminated is less than 5, leave the retained digits unchanged.

The following examples illustrate the rule for rounding off:

1. 0.1414 rounded to thousandths is 0.141.
2. 3.147 rounded to tenths is 3.1.
3. 475 rounded to the nearest hundred is 500.

Observe carefully that the answer to example 2 is not 3.2. Some trainees make the error of treating the rounding process as a kind of chain reaction, in which one first rounds 3.147 to 3.15 and then rounds 3.15 to 3.2. The error of this method is apparent when we note that 147/1000 is closer to 100/1000 than it is to 200/1000.

Problems of the following type are sometimes confusing: Reduce 2.998 to the nearest hundredth. To drop the end figure we must increase the next figure by 1. The final result is 3.00. We retain the zeros to show that the answer is carried to the nearest hundredth.

Practice problems. Round off as indicated:

1. 0.5862 to hundredths
2. 0.345 to tenths
3. 2346 to hundreds
4. 3.999 to hundredths

Answers:

1. 0.59	3. 2300
2. 0.3	4. 4.00

CHANGING DECIMALS TO COMMON FRACTIONS

Any decimal may be reduced to a common fraction. To do this we simply write out the numerator and denominator in full and reduce to lowest terms. For example, to change 0.12 to a common fraction, we simply write out the fraction in full,

$$\frac{12}{100}$$

and reduce to lowest terms,

$$\frac{\overset{3}{\cancel{12}}}{\underset{25}{\cancel{100}}} = \frac{3}{25}$$

Likewise, 0.77 is written

$$\frac{77}{100}$$

but this is in lowest terms so the fraction cannot be further reduced.

One way of checking to see if a decimal fraction can be reduced to lower terms is to consider the makeup of the decimal denominator. The denominator is always 10 or a power of 10. Inspection shows that the prime factors of 10 are 5 and 2. Thus, the numerator must be divisible by 5 or 2 or both, or the fraction cannot be reduced.

EXAMPLE: Change the decimal 0.0625 to a common fraction and reduce to lowest terms.

SOLUTION: $0.0625 = \frac{625}{10000}$

$$= \frac{625 \div 25}{10000 \div 25} = \frac{25}{400}$$

$$= \frac{1}{16}$$

Complex decimals are changed to common fractions by first writing out the numerator and denominator in full and then reducing the resulting complex fraction in the usual way. For example, to reduce $0.12\frac{1}{2}$, we first write

$$\frac{12\frac{1}{2}}{100}$$

Writing the numerator as an improper fraction we have

$$\frac{\frac{25}{2}}{100}$$

and applying the reciprocal method of division, we have

$$\frac{\overset{1}{\cancel{25}}}{2} \times \frac{1}{\underset{4}{\cancel{100}}} = \frac{1}{8}$$

Practice problems. Change the following decimals to common fractions in lowest terms:

1. 0.25
2. 0.375
3. $0.6\frac{1}{4}$
4. $0.03\frac{1}{5}$

Answers:

1. 1/4
2. 3/8
3. 5/8
4. 4/125

CHANGING COMMON FRACTIONS TO DECIMALS

The only difference between a decimal fraction and a common fraction is that the decimal fraction has 1 with a certain number of zeros (in other words, a power of 10) for a denominator. Thus, a common fraction can be changed to a decimal if it can be reduced to a fraction having a power of 10 for a denominator.

If the denominator of the common fraction in its lowest terms is made up of the prime factors 2 or 5 or both, the fraction can be converted to an exact decimal. If some other prime factor is present, the fraction cannot be converted exactly. The truth of this is evident when we consider the denominator of the new fraction. It must always be 10 or a power of 10, and we know the factors of such a number are always 2's and 5's.

The method of converting a common fraction to a decimal is illustrated as follows:

EXAMPLE: Convert 3/4 to a decimal.

SOLUTION: $\frac{3}{4} = \frac{300}{400}$

$$= \frac{300}{4} \times \frac{1}{100}$$

$$= 75 \times \frac{1}{100}$$

$$= 0.75$$

Notice that the original fraction could have been rewritten as 3000/4000, in which case the result would have been 0.750. On the other hand, if the original fraction had been rewritten as 30/40, the resulting division of 4 into 30 would not have been possible without a remainder. When the denominator in the original fraction has only 2's and 5's as factors, so that we know a remainder is not necessary, the fraction should be rewritten with enough 0's to complete the division with no remainder.

Observation of the results in the foregoing example leads to a shortcut in the conversion method. Noting that the factor 1/100 ultimately enters the answer in the form of a decimal, we could introduce the decimal point as the final step without ever writing the fraction 1/100. Thus the rule for changing fractions to decimals is as follows:

1. Annex enough 0's to the numerator of the original fraction so that the division will be exact (no remainder).
2. Divide the original denominator into the new numerator formed by annexing the 0's.
3. Place the decimal point in the answer so that the number of decimal places in the answer is the same as the number of 0's annexed to the original numerator.

If a mixed number in common fraction form is to be converted, convert only the fractional part and then write the two parts together. This is illustrated as follows:

$$2\frac{3}{4} = 2 + \frac{3}{4} = 2 + .75 = 2.75$$

Practice problems. Convert the following common fractions and mixed numbers to decimal form:

1. $\frac{1}{4}$ 2. $\frac{3}{8}$ 3. $\frac{5}{32}$ 4. $2\frac{5}{16}$

Answers:

1. 0.25 2. 0.375 3. 0.15625 4. 2.3125

Nonterminating Decimals

As stated previously, if the denominator of a common fraction contains some prime factor other than 2 or 5, the fraction cannot be converted completely to a decimal. When such fractions are converted according to the foregoing rule, the decimal resulting will never terminate. Consider the fraction 1/3. Applying the rule, we have

$$\begin{array}{r} .333\ \cdots \\ 3\overline{)1.0000} \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \end{array}$$

The division will continue indefinitely. Any common fraction that cannot be converted exactly yields a decimal that will never terminate and in which the digits sooner or later recur. In the previous example, the recurring digit was 3. In the fraction 5/11, we have

$$\begin{array}{r} .4545 \\ 11\overline{)5.0000} \\ \underline{4\,4} \\ 60 \\ \underline{55} \\ 50 \\ \underline{44} \\ 60 \\ \underline{55} \end{array}$$

The recurring digits are 4 and 5.

When a common fraction generates such a repeating decimal, it becomes necessary to arbitrarily select a point at which to cease the repetition. This may be done in two ways. We may write the decimal fraction by rounding off at the desired point. For example, to round off the decimal generated by $\frac{1}{3}$ to hundredths, we carry the division to thousandths, see that this figure is less than 5, and drop it. Thus, $\frac{1}{3}$ rounded to hundredths is 0.33. The other method is to carry the division to the desired number of decimal places and carry the remaining incomplete division as a common fraction—that is, we write the result of a complex decimal. For example, $\frac{1}{3}$ carried to thousandths would be

$$\frac{1}{3} = \begin{array}{r} .333\frac{1}{3} \\ 3\overline{)1.000}\phantom{\frac{1}{3}} \\ \underline{9}\phantom{\frac{1}{3}} \\ 10\phantom{\frac{1}{3}} \\ \underline{9}\phantom{\frac{1}{3}} \\ 10\phantom{\frac{1}{3}} \\ \underline{9}\phantom{\frac{1}{3}} \\ 1\phantom{\frac{1}{3}} \end{array}$$

Practice problems. Change the following common fractions to decimals with three places and carry the incomplete division as a common fraction:

1. $\frac{7}{13}$ 2. $\frac{5}{9}$ 3. $\frac{4}{15}$ 4. $\frac{5}{12}$

Answers:

1. $0.538\frac{6}{13}$ 3. $0.266\frac{2}{3}$

2. $0.555\frac{5}{9}$ 4. $0.416\frac{2}{3}$

OPERATION WITH DECIMALS

In the study of addition of whole numbers, it was established that units must be added to units, tens to tens, hundreds to hundreds, etc. For convenience, in adding several numbers, units were written under units, tens under tens, etc. The addition of decimals is accomplished in the same manner.

ADDITION

In adding decimals, tenths are written under tenths, hundredths under hundredths, etc. When this is done, the decimal points fall in a straight line. The addition is the same as in adding whole numbers. Consider the following example:

$$\begin{array}{r} 2.18 \\ 34.35 \\ 0.14 \\ \underline{4.90} \\ 41.57 \end{array}$$

Adding the first column on the right gives 17 hundredths or 1 tenth and 7 hundredths. As with whole numbers, we write the 7 under the hundredths column and add the 1 tenth in the tenths column—that is, the column of the next higher order. The sum of the tenths column is 15 tenths or 1 unit and 5 tenths. The 5 is written under the tenths column and the 1 is added in the units column.

It is evident that if the decimal points are kept in a straight line—that is, if the place values are kept in the proper columns—addition with decimals may be accomplished in the ordinary manner of addition of whole numbers. It should also be noted that the decimal point of the sum falls directly under the decimal points of the addends.

SUBTRACTION

Subtraction of decimals likewise involves no new principles. Notice that the place values of the subtrahend in the following example are fixed directly under the corresponding place values in the minuend. Notice also that this causes the decimal points to be alined and that the figures in the difference (answer) also retain the correct columnar alinement.

$$\begin{array}{r} 45.76 \\ \underline{-31.87} \\ 13.89 \end{array}$$

We subtract column by column, as with whole numbers, beginning at the right.

Practice problems. Add or subtract as indicated:

1. 12.3 + 2.13 + 4 + 1.234
2. 0.5 + 0.04 + 12.001 + 10
3. 237.5 - 217.9
4. 9.04 - 7.156

Answers:

1. 19.664
2. 22.541
3. 19.6
4. 1.884

MULTIPLICATION

Multiplication of a decimal by a whole number may be explained by expressing the decimal as a fraction.

EXAMPLE: Multiply 6.12 by 4.

SOLUTION: $$\frac{4}{1} \times \frac{612}{100} = \frac{2448}{100}$$
$$= 24.48$$

When we perform the multiplication keeping the decimal form, we have

$$\begin{array}{r} 6.12 \\ \underline{4} \\ 24.48 \end{array}$$

By common sense, it is apparent that the whole number 4 times the whole number 6, with some fraction, will yield a number in the neighborhood of 24. Hence, the placing of the decimal point is reasonable.

An examination of several examples will reveal that the product of a decimal and a whole number has the same number of decimal places as the factor containing the decimal. Zeros, if any, at the end of the decimal should be rejected.

Multiplication of Two Decimals

To show the rule for multiplying two decimals together, we multiply the decimal in fractional form first and then in the conventional way, as in the following example:

$$0.4 \times 0.37$$

Writing these decimals as common fractions, we have

$$\frac{4}{10} \times \frac{37}{100} = \frac{4 \times 37}{10 \times 100}$$
$$= \frac{148}{1000}$$
$$= 0.148$$

In decimal form the problem is

$$\begin{array}{r} 0.37 \\ \underline{0.4} \\ 0.148 \end{array}$$

The placing of the decimal point is reasonable, since 4 tenths of .37 hundredths is a little less than half of 37 hundredths, or about 15 hundredths.

Consider the following example:

$$4.316 \times 3.4$$

In the common fraction form, we have

$$\frac{4316}{1000} \times \frac{34}{10} = \frac{4316 \times 34}{1000 \times 10} = \frac{146744}{10000} = 14.6744$$

In the decimal form the problem is

$$\begin{array}{r} 4.316 \\ \underline{3.4} \\ 17264 \\ \underline{12948} \\ 14.6744 \end{array}$$

We note that 4 and a fraction times 3 and a fraction yields a product in the neighborhood of 12. Thus, the decimal point is in the logical place.

In the above examples it should be noted in each case that when we multiply the decimals together we are multiplying the numerators. When we place the decimal point by adding the number of decimal places in the multiplier and multiplicand, we are in effect multiplying the denominators.

When the numbers multiplied together are thought of as the numerators, the decimal points may be temporarily disregarded and the numbers may be considered whole. This justifies the apparent disregard for place value in the multiplication of decimals. We see that the rule for multiplying decimals is only a modification of the rule for multiplying fractions.

To multiply numbers in which one or more of the factors contain a decimal, multiply as though the numbers were whole numbers. Mark off as many decimal places in the product as there are decimal places in the factors together.

Practice problems. Multiply as indicated:

1. 3.7 x 0.02
2. 0.45 x 0.7
3. $\begin{array}{r} 6.5 \\ \underline{\times 0.01} \end{array}$
4. $\begin{array}{r} 0.0073 \\ \underline{\times 5.4} \end{array}$

Answers:

1. 0.074
2. 0.315
3. 0.065
4. 0.03942

Multiplying by Powers of 10

Multiplying by a power of 10 (10, 100, 1,000, etc.) is done mechanically by simply moving the decimal point to the right as many places as there are zeros in the multiplier. For example, 0.00687 is multiplied by 1,000 by moving the decimal point three places to the right as follows:

$$1{,}000 \times 0.00687 = 6.87$$

Multiplying a number by 0.1, 0.01, 0.001, etc., is done mechanically by simply moving the decimal point to the left as many places as there are decimal places in the multiplier. For example, 348.2 is multiplied by 0.001 by moving the decimal point three places to the left as follows:

$$348.2 \times 0.001 = 0.3482$$

DIVISION

When the dividend is a whole number, we recognize the problem of division as that of converting a common fraction to a decimal. Thus in the example 5 ÷ 8, we recall that the problem could be written

$$\frac{5000}{1000} \div 8 = \frac{5000 \div 8}{1000} = \frac{625}{1000} = .625$$

This same problem may be worked by the following, more direct method:

$$\frac{5}{8} = \begin{array}{r} .625 \\ 8\overline{)5.000} \\ \underline{4\ 8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \end{array}$$

Since not all decimals generated by division terminate so early as that in the above example, if at all, it should be predetermined as to how many decimal places it is desired to carry the quotient. If it is decided to terminate a quotient at the third decimal place, the division should be carried to the fourth place so that the correct rounding off to the third place may be determined.

When the dividend contains a decimal, the same procedure applies as when the dividend is whole. Notice the following examples (rounded to three decimal places):

1. 6.31 ÷ 8

```
     .7887 = .789
  8/6.3100
    5 6
      71
      64
       70
       64
        60
        56
         4
```

2. 0.0288 ÷ 32

```
      0.0009 = 0.001
  32/0.0288
        288
```

Observe in each case (including the case where the dividend is whole), that the quotient contains the same number of decimal places as the number used in the dividend. Notice also that the place values are rigid; that is, tenths in the quotient appear over tenths in the dividend, hundredths over hundredths, etc.

Practice problems. In the following division problems, round off each quotient correct to three decimal places.

1. 10 ÷ 6	3. 2.743 ÷ 77
2. 23.5 ÷ 16	4. 1.00 ÷ 3

Answers:

1. 1.667	3. 0.036
2. 1.469	4. 0.333

Decimal Divisors

In the foregoing examples, the divisor in each case was an integer. Division with divisors which are decimals may be accomplished by changing the divisor and dividend so that the divisor becomes a whole number.

Recalling that every division expression may be written in fraction form, we use the fundamental rule of fractions as follows: Rewrite the division problem as a fraction. Multiply the numerator (dividend) and denominator (divisor) by 10, 100, or some higher power of 10; the power of 10 must be large enough to change the divisor to a whole number. This rule is illustrated as follows:

$$2.568 \div 0.24 = \frac{2.568}{0.24}$$

$$= \frac{2.568}{0.24} \times \frac{100}{100}$$

$$= \frac{256.8}{24}$$

Thus 2.568 divided by 0.24 is the same as 256.8 divided by 24.

From the mechanical standpoint, the foregoing rule has the effect of moving the decimal point to the right, as many places as necessary to change the divisor to an integer. Therefore the rule is sometimes stated as follows: When the divisor is a decimal, change it to a whole number by moving the decimal point to the right. Balance the change in the divisor by moving the decimal point in the dividend an equal number of places to the right.

The following example illustrates this version of the rule:

```
         9 1.1
  0.9∧/81.9∧9
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The inverted v, called a caret, is used as a marker to indicate the new position of the decimal point. Notice that the decimal point in the quotient is placed immediately above the caret in the dividend. Alinement of the first quotient digit immediately above the 1 in the dividend, and the second quotient digit above the 9, assures that these digits are placed properly with respect to the decimal point.

Practice problems. In the following division problems, round off each quotient to three decimal places:

1. 0.02958 ÷ 0.12	3. 4610 ÷ 0.875
2. 30.625 ÷ 3.5	4. 0.000576 ÷ 0.008

Answers:

1. 0.247	3. 5268.571
2. 8.750	4. 0.072

Dividing by Powers of 10

Division of any number by 10, 100, 1,000, etc., is really just an exercise in placing the decimal point of a decimal fraction. Thus, 5,031 ÷ 100 may be thought of as the decimal fraction $\frac{5031}{100}$; to remove the denominator, we simply count off two places from the right. Thus,

$$\frac{5031}{100} = 50.31$$

The following three examples serve to illustrate this procedure further:

$$401 \div 10 = 40.1$$

$$2 \div 1{,}000 = .002$$

$$11{,}431 \div 100 = 114.31$$

If the dividend already contains a decimal part, begin counting with the first number to the left of the decimal point. Thus, 243.6 ÷ 100 = 2.436. When the decimal point is not shown in a number, it is always considered to be to the right of the right-hand digit.

Dividing by 0.1, 0.01, 0.001, etc., may also be accomplished by a simple mechanical rule. We simply begin at the position of the decimal point in the dividend and count off as many places to the right as there are decimal places in the divisor. The decimal point is then placed to the right of the last digit counted. If there are not enough digits, zeros may be added.

The foregoing rule is based on the fact that 0.1 is really $\frac{1}{10}$, 0.01 is $\frac{1}{100}$, 0.001 is $\frac{1}{1000}$, etc. For example,

$$\begin{aligned} 23 \div 0.1 &= 23 \div \frac{1}{10} \\ &= 23 \times \frac{10}{1} \\ &= 230 \end{aligned}$$

Notice that dividing by 0.1 is the same as multiplying by 10. Likewise,

$$\begin{aligned} 234.1 \div 0.001 &= 234.1 \div \frac{1}{1000} \\ &= 234.1 \times \frac{1000}{1} \\ &= 234{,}100 \end{aligned}$$

and

$$24 \div 0.01 = 24 \div \frac{1}{100} = 24 \times \frac{100}{1} = 2{,}400$$

Practice problems. Divide by relocation of the decimal point.

1. 276 ÷ 100
2. 2,845 ÷ 1,000
3. 276 ÷ 0.01
4. 2,845 ÷ 0.001

Answers:

1. 2.76
2. 2.845
3. 27,600
4. 2,845,000